ASSIGNMENT

Q1/ Given:- $V(x) = \begin{cases} \frac{1}{2} m\omega^2 \left(x - \frac{a}{2}\right)^2 & \text{for } 0 < x < a \\ \infty & \text{otherwise} \end{cases}$

Ans/

$$\hat{H}' = V(x) = \frac{1}{2} m\omega^2 \left(x - \frac{a}{2}\right)^2$$

$$E_n' = \langle n^0 | H' | n^0 \rangle$$

$$n^0 = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

$$E_n' = \int_0^L \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \frac{1}{2} m\omega^2 \left(x - \frac{a}{2}\right)^2 \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) dx$$

$$E_n' \Rightarrow \langle n^0 | H' | n^0 \rangle = \langle n^0 | V_0 | n^0 \rangle$$
$$= V_0 (n^0 | n^0) = V_0 = V(x)$$

$$E_n = E_n^0 + E_n'$$
$$\Rightarrow \frac{\hbar^2}{8mL^2} n^2 + V(x)$$
$$\Rightarrow \frac{\hbar^2}{8mL^2} n^2 + \frac{1}{2} m\omega^2 \left(x - \frac{a}{2}\right)^2$$

1(b):- Standard deviation

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 |v(x)|^2 dx$$

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2 \left[\left(\frac{1}{2} m\omega^2 \left(x - \frac{a}{2}\right)^2\right)\right]^2 dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{4} m^2\omega^2 \left(x - \frac{a}{2}\right)^4 dx \, (x^2)$$

$$\Rightarrow \frac{1}{4} m^2\omega^2 \int_{-\infty}^{\infty} \left(x - \frac{a}{2}\right)^4 x^2 dx$$

$$\Rightarrow \frac{1}{4} m^2\omega^2 \left[\frac{(x-a)^5 (15x^2 + 5ax + a^2)}{105}\right]_{-\infty}^{\infty}$$

$$\Delta x^2 = \langle x^2\rangle - \langle x\rangle^2$$

$$\Rightarrow \frac{1}{4} m^2\omega^2 \left[\frac{(x-a)^5 (15x^2 + 5ax + a^2)}{105}\right]_{-\infty}^{\infty} -$$

$$\frac{1}{4} m^2\omega^2 \left[\frac{x^3 (6x^2 - 15a/2\, x + 10a^2/4)}{30}\right]$$

$$\Delta x \Rightarrow \hbar \sqrt{\frac{1}{4} m^2\omega^2} \Rightarrow \frac{\hbar}{2} m\omega$$

$$\boxed{\Delta x = \frac{\hbar}{2} m\omega}$$

1(c): Position

$$\langle x\rangle = \int_{-\infty}^{\infty} x \left| v(x) \right| dx$$

$$\langle x\rangle = 0$$

Standard deviation

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

$$\Delta x \Rightarrow \sqrt{0 - \langle x\rangle^2}$$

$$\boxed{\Delta x \Rightarrow \sqrt{\frac{\hbar}{2m\omega}}}$$

1(d) the Δx by part b is $\frac{\hbar}{2} m\omega$

and by part c is $\sqrt{\frac{\hbar}{2m\omega}}$, the value of

SD by part (c) will be smaller than part b.

the relative size will be $\frac{\hbar}{2}(m\omega) - \sqrt{\frac{\hbar}{2}\frac{m\omega}{m^2\omega^2}}$

$$R.S. \Rightarrow \frac{\hbar}{2} m\omega - \sqrt{\frac{\hbar}{2m\omega}}$$

Q2 $\quad J_\pm f_j^m = \hbar\sqrt{j \mp m(j \mp m+1)}\, f_j^{m\pm 1}$

Ans 2(a): $\langle j_\pm f_j^m\rangle \Rightarrow j_x^n [j \pm f_i^m)$

$$= \hbar^2 \sqrt{(j \mp m)(j \pm f_i)}$$

$$\langle j_\pm f_j^m\rangle = i\hbar \iiint (j_x^2 + j_y^2 + j_z^2)\left(j_x \frac{\partial f}{\partial x} - z\frac{\partial f}{\partial y} - z\frac{\partial f}{\partial z}\right) dx\,dy\,dz$$

$$\Rightarrow \quad \langle j \pm f; m \rangle = \langle j_x \pm j_y \pm j_z | f^m \rangle$$

$$j_x \pm i j_y \pm i j_z = j \pm$$

$$\langle j_x \pm f; m | j_y \pm f; m \rangle + j_z \pm f; m \rangle = (A; m)^2 \langle f; m \pm 1 | f; m \pm 1 \rangle$$

$$A; m = \hbar \sqrt{(j_x(j_x+1) - m^2 \mp m) + (j_y(j_y+1) - m^2 \mp m) + (j_z(j_z+1) - m^2 \mp m)}$$

2(b):- there are 6 such states available

Six basis states given here

$$|1\rangle|\uparrow\rangle, \quad |1\rangle|\downarrow\rangle,$$
$$|0\rangle|\uparrow\rangle, \quad |0\rangle|\downarrow\rangle,$$
$$|-1\rangle|\uparrow\rangle, \quad |-1\rangle|\downarrow\rangle$$

in terms of normal states

$$A_1^m \Rightarrow \hbar \sqrt{(1+m) - m^2 \mp m}$$

$$A_2^m \Rightarrow \hbar \sqrt{(-1+m) - m^2 \pm m}$$

$$A_3^m \Rightarrow \hbar \sqrt{m - m^2 \mp m}$$

$$A_4^m = \hbar \sqrt{-m - m^2 \mp m}$$

$$A_5^m \Rightarrow \hbar \sqrt{-1 - m^2 \mp m}$$

$$A_6^m \Rightarrow \hbar \sqrt{1 - m^2 \mp m}$$

2(c):- the normalise states having value 1 with clockwise spin shows the state of condition having moment j_z.

the normalise states having value 0 with anti clock wise state shows the state of condition having moment $\hat{j}_z^2$.

2(d):- orthogonalise and normalised states:-

$$|\psi\rangle \Rightarrow \lambda|1\rangle + \mu|-1\rangle$$

$$\langle\psi| \Rightarrow \lambda|-1\rangle + \mu|0\rangle$$

$$|\phi\rangle = \alpha|\langle 0| + \beta|0\rangle$$

$$\langle\phi| = \alpha^*\langle 0| + \beta^*\langle 0|$$

$$\langle\chi| \Rightarrow m\langle -1| + n|-1\rangle$$

$$|\chi\rangle = m^*\langle -1| + n^*|-1\rangle$$

the states with negative values corresponds to the states momentum of J_z and all positive values corresponds to the state momentum of J.

3. Given:- $H = p^2/2m + (k/2)x^2$

As 3(a):- Consider schrodinger operators $\hat{x}$ and $\hat{p}$.

$$\frac{d}{dt}\hat{x}_H(t) = \frac{1}{i\hbar}\frac{\hat{p}_H(t)}{m}i\hbar = \frac{\hat{p}_H(t)}{m}$$

$$\frac{d}{dt}\hat{x}_H(t) = \frac{\hat{p}_H(t)}{m}$$

momentum operator

$$\frac{d}{dt}\hat{p}_H(t) = \frac{1}{i\hbar}[\hat{p}_H(t), H_H(t)]$$

$$\Rightarrow -m\omega^2\hat{x}_H(t)$$

$$\Rightarrow -m\omega\hat{A}\sin\omega t + m\hat{B}\cos\omega t$$

3(b) For $p^3|n\rangle$

$\hat{x}_H(3) = \hat{A} = \hat{x}$, $p_H(3) = m\omega\hat{B} = \hat{p}$.

$\hat{p}^3|n\rangle \Rightarrow \hat{p}\cos\omega t - m\omega\hat{x}\sin\omega t$.

3(c):- $\langle p^j\rangle$ Given:-

$\frac{d}{dt}\hat{p}_H(t) \Rightarrow -m\omega\hat{A}\sin\omega t + m\omega\hat{B}\cos\omega t$

$$\langle p^j\rangle \Rightarrow \frac{1}{2m}\langle\hat{p}\cos\omega t - m\omega\hat{x}\sin\omega t\rangle^2 + \frac{1}{2}m\omega^2\left(-m\omega\hat{A}\sin\omega t + \frac{m\omega\hat{B}\cos\omega t}{m\omega}\right)^2$$

$$\langle p^j\rangle \Rightarrow \frac{1}{2m}\langle\hat{p}\cos\omega t - m\omega\hat{x}\sin\omega t\rangle^2 + \frac{1}{2}m\omega^2\left(-m\omega\hat{A}\sin\omega t + \frac{m\omega\hat{B}\cos\omega t}{m\omega}\right)^2$$

3(d):- it makes sense physically because all values that are required to define are present.

$$\langle x^j\rangle = \int_{-\infty}^{\infty} x^j |H|^j dx$$

(4) (a):- A) $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial y^2}(y,t)$

$$+U(x)\psi(x,t)\psi(y,t) = i\hbar\frac{\partial\psi}{\partial t}(x,y,t)$$

$$e^{i(Kx-\omega t)} = e^{iKx} e^{-i\omega t}$$

$$\psi(x,t) = \psi(x)\,\phi(t)$$

$$\psi(y,t) = \psi(y)\,\phi(t)$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial y^2} + U(x)\,\psi(x)\,\psi(y)$$

$$\Rightarrow E(\psi x) + E\psi(y)$$

4 (b):- If variables are seperable the equation for x

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{\hbar^2}{2m}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dR(r)}{dr}\right) - \frac{m^2}{r^2}R(r)\right] + \frac{1}{8}m\omega_1^2 r^2 R(r) = E(\psi(x))$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial y^2} = -\frac{\hbar^2}{2m}\left[r\frac{d^2}{dr^2}\left(r^2\frac{dR(r)}{dr}\right) - \frac{m^2}{r^2}(R(r))^{-1}\right] + \frac{1}{8}\left[m\omega_1^2 r^2 R(r)\right]$$

$$= E(\psi(y), r)$$

~~boundary~~

4(c):- boundary condition:-

$y=0, \quad \varphi=0, \quad r=0$

$y=r=\varphi=\varphi(y,r)$,,

4(d):- for $y=0$,

$$-\frac{\hbar^2}{2m}\left[r^2\frac{d^2}{dr^2}\left(r^2\frac{dR(r)}{dr}\right)-\frac{m^2}{r^2}(R(r))^{-1}\right]$$
$$+\frac{1}{8}m\omega_1^2 r^2 R(r) = E(r)$$

for $y=0$, $\quad E(r)=0$

for $y=r=$ $$-\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial r^2}-\frac{\hbar^2}{2m}\left[r\frac{\partial^2}{\partial r^2}\left(r^2\frac{\partial R(r)}{\partial r}\right)\right.$$
$$\left.-\frac{m^2}{r^2}(R(r))^{-1}\right]+\frac{1}{8}\left[m\omega_1^2 r^2 R(r)\right]$$
$$=E\,(\varphi(r),r)$$

4(e):- when system is cut energy condition

$$\Rightarrow E[\varphi(L),r] \Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial r^2}-\frac{\hbar^2}{2m}$$
$$\left[L\frac{\partial^2}{\partial L^2}\left(r^2\frac{\partial L(r)}{\partial L}\right)\right]-\frac{m^2}{r^2}(R(L))$$
$$+\frac{1}{8}m\omega^2\frac{r^2}{L}R(L)$$

boundary

4(c):- boundary condition:-

$y=0, \quad \varphi=0, \quad r=0$

$y=r \Rightarrow \varphi=\varphi(y,r)$,,

4(d):- for $y=0$,

$$-\frac{\hbar^2}{2m}\left[r^2\frac{d^2}{dr^2}\left(r^2\frac{dR(r)}{dr}\right)-\frac{m^2}{r^2}(R(r))^{-1}\right]$$
$$+\frac{1}{8}m\omega_1^2 r^2 R(r) = E(r)$$

for $y=0$, $E(r)=0$

for $y=r$ $=$ $-\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial r^2}-\frac{\hbar^2}{2m}\left[r\frac{\partial^2}{\partial r^2}\left(r^2\frac{\partial R(r)}{\partial r}\right)\right.$
$$\left.-\frac{m^2}{r^2}(R(r))^{-1}\right]+\frac{1}{8}\left[m\omega_1^2 r^2 R(r)\right]$$
$$= E\,(\varphi(r), r)$$

4(e):- when system is cut energy condition

$$\Rightarrow E[\varphi(L), r] \Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial r^2} - \frac{\hbar^2}{2m}$$
$$\left[L\frac{\partial^2}{\partial L^2}\left(L^2\frac{\partial L(r)}{\partial L}\right)\right] - \frac{m^2}{r^2}(R(L))$$
$$+\frac{1}{8}m\omega^2\frac{r^2}{L}R(L)$$

4(f) boundary condition

$x=0$, $E\,\psi(L)=0$, $L=0$

$x=r$: $E[\psi(L)] = \psi(r)$

$x=L$ = $E[\psi(L)] \Rightarrow \psi(L)+r$

5(a):- $V=L^3$

$$N \Rightarrow V\int \frac{d^3k}{(2\pi)^3} = V\int_0^{k_F} \frac{4\pi}{(2\pi)^3} k^3\, dk$$

$$\frac{Vk_F^3}{6\pi^2} \Rightarrow \frac{L^3 k_F^3}{6\pi^2}$$

$$k_F = (3\pi^2 n)^{1/3}$$

$$E_F = 2V\int_{k<V} \frac{\hbar^2 k^2}{2m}\frac{d^3k}{(2\pi)^3} = 2L^3\frac{\hbar^2}{2m}\frac{4\pi}{5(2\pi)^3}k_F^5$$

$$= 2L^3\frac{\hbar^2}{2m}\frac{4\pi}{40\pi^3}(3\pi^2 n)^{5/3}$$

5(b): Total energy =

$E' = E_F$ + Pressure energy

$$2L^3\frac{\hbar^2}{2m}\frac{4\pi}{40\pi^3}(3\pi^2 n)^{5/3} + \frac{1}{PV}$$

$$E' = 2L^3\frac{\hbar^2}{2m}\frac{4\pi}{40\pi^3}(3\pi^2 n)^{5/3} + \frac{1}{PL^3}$$

S(c) :- energy in P_x

$$P_x = \frac{\partial F_x}{\partial N_e}\Big|_{T.V.} = K_B \frac{E_t}{h} \ln (n_e \lambda_e)^3$$

$$P_y = \frac{\partial F_y}{\partial N_e}\Big|_{T.V} = K_B \frac{E_t}{h} K_B (n_e \lambda_e)^3$$

$$P_z = \frac{\partial F_z}{\partial N_e}\Big|_{T.V.} = K_B \frac{E_t}{h} K_B (n_e \lambda_e)^3$$

energy

$$E_x = CP_x = C_x K_B \frac{E_t}{h} \ln (n_e \lambda_e)^3$$

$$E_y = CP_y = C_y K_B \frac{E_t}{h} \ln (n_e \lambda_e)^3$$

$$E_z = CP_z = C_z K_B \frac{E_t}{h} \ln (n_e \lambda_e)^3$$

S(d) :- when $n = N/4$, $\varepsilon = PC$

$$D(\varepsilon)d\varepsilon = \frac{2V}{h^3} 4\pi p^2 dp = \frac{V}{\pi^2 h^2 c^2} \varepsilon^2 d\varepsilon$$

$$N = \int_0^{\varepsilon_F} \frac{V}{\pi^2 h^2 c^3} \varepsilon^3 d\varepsilon = \frac{V}{3\pi^2 h^2 c^3} \varepsilon_F^3$$

$$\varepsilon^3 = h\pi c \left(\frac{3n}{\pi}\right)$$

$$c = 3\sqrt{h\pi c \left(\frac{3n}{\pi}\right)}$$

5(e)! - total energy = $U_0 = A\int_0^{\epsilon_F} \epsilon^2 d\epsilon = \frac{A}{4}\epsilon_F^4$

$N =$

5(e) total energy $\int_0^{\epsilon_F} \epsilon D(\epsilon) d\epsilon$, $D\epsilon = A\epsilon^2$

$$U_0 = \frac{A}{N}\int_0^{\epsilon_F} \epsilon^3 d\epsilon = \frac{A}{N(4)}\epsilon_F^4$$

Pressure = $\frac{U_0}{V} = \frac{U_0}{L^3}$

$$P = \frac{A}{4NL^3}\epsilon_F^4$$

5(f)! - the system is closed.

the system is in equillibrium

In terms of m

$m_{in} = m_{out}$

$m_{excited} = m_{lower}$

and for $n = N/V$

$n_1 = n_2$

$\frac{N_1}{V_1} = \frac{N_2}{V_2}$